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A strongly degenerate parabolic equation in gas filtration problems

Pasquale Ambrosio, Salvatore Cuomo, Mariapia De Rosa, Antonia Passarelli di Napoli
University of Naples "Federico II", Italy

Motivated by applications to gas filtration problems, and therefore assuming that $n = 2$ or $n = 3$, we first want to verify if there exists a (unique) numerical solution $\tilde{u}$ to the Cauchy-Dirichlet problem

$$(1) \qquad \begin{cases} \partial_t u - \operatorname{div}\left((|Du| - 1)_+ \, \frac{Du}{|Du|}\right) = f & \text{in } \Omega \times (0,T) \\ u = g & \text{on } \partial\Omega \times (0,T) \\ u = h & \text{on } \Omega \times \{0\}\,, \end{cases}$$

once the domain $\Omega \subset \mathbb{R}^n$ and the initial-boundary conditions have been chosen in a suitable and significant way and evaluate its qualitative features.
After introducing a suitable family $\{\mathcal{P}_\varepsilon\}_{\varepsilon>0}$ of Cauchy-Dirichlet problems which approximate (1) in a certain sense as $\varepsilon \to 0$, we aim to understand whether the numerical solution $\tilde{u}_\varepsilon$ of $\mathcal{P}_\varepsilon$ converges in a suitable sense to the solution $\tilde{u}$ when ε approaches zero.

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E–mail: `pasquale.ambrosio2@unina.it`.

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WENO-based quasi-interpolation in the Bernstein basis and applications

F. Aràndiga
Departament de Matèematiques, Universitat de València, Spain
D. Barrera
Department of Applied Mathematics, University of Granada, Spain
S. Eddargani
Department of Mathematics, University of Rome Tor Vergata, Italy
M. J. Ibáñez
Department of Applied Mathematics, University of Granada, Spain
J. B. Roldán
Department of Electronics, University of Granada, Spain

Resistive Random-Access Memories are devices that are being successfully used in the industry for non-volatile memory applications and they show promising possibilities to overcome the hurdles of the technologies employed nowadays for future applications such as neuromorphic computing. The hardware implementations of neural networks allow the generated results of the computation to be stored on-site without data shuffing in and out, therefore avoiding the drawbacks associated with the so-called Von Neumann's bottleneck [1]. The operation both in the memory and in the neuromorphic computing framework is based on voltage pulses as input signals. Therefore, procedure for constructing accurate approximants is necessary, which takes into account the discontinuity of the functions involved. In addition, the number of measured data is generally very large, so the approximation method must have a moderate computational cost. We propose the construction of quasi-interpolating splines defined directly in the Bernstein basis relative to each of the intervals induced by the partition [2]. Since it is not uniform, the masks used to define the Béezier ordinates will depend on the specific interval we are working on. The construction is combined with a WENO method [3] to get quasi-interpolants that fit the measured data well.

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E–mail: dbarrera@ugr.es.

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Continuous (semi-)frames for unbounded operators

Giorgia Bellomonte
Dipartimento di Matematica e Informatica, Università degli Studi di Palermo, I-90123 Palermo, Italy

A generalization of the notion of A-frame [4] to the continuous setting and for a densely defined and possibly unbounded operator A on a Hilbert space $\mathcal{H}$ (with domain $\mathcal{D}(A)$) has been introduced and studied in [2]. Equivalent formulations in terms of atomic systems, existence results and some characterizations are given. As it is known, if the operator A is bounded in $\mathcal{H}$, then every element Af in $\mathcal{R}(A)$, the range of A, can be decomposed as a combination of a family of vectors (the elements of an A-frame, which are a Bessel family and do not necessarily belong to $\mathcal{R}(A)$) with coefficients continuously depending on f, see e.g. [4] and [5]. On the contrary, the unboundedness of A leads to the fact that, in a similar decomposition of the elements in $\mathcal{R}(A)$, the coefficients can not depend continuously on f. In [2] this problem is addressed in two ways, going over what have been done in the discrete case in [3]. In one case, a non-Bessel family and coefficients depending continuously on $f \in \mathcal{D}(A)$ have been considered, in another one a Bessel family and coefficients depending continuously on $f \in \mathcal{D}(A)$ only in the graph topology of A are taken, to exploit the results on bounded operators between different Hilbert spaces. Lastly, the notion of lower semi-frame controlled by a densely defined operator A or, for short, a *weak lower A-semi-frame* is introduced and studied. In particular, a comparison with that one of lower atomic systems is made. Duality properties are discussed and several possible definitions for weak A-upper semi-frames are suggested.

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E–mail: address@server.it.

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Approximation of set-valued functions by metric integral operators

Elena E. Berdysheva
Department of Mathematics and Applied Mathematics, University of Cape Town
Nira Dyn, Elza Farkhi
School of Mathematical Sciences, Tel-Aviv University
Alona Mokhov
Afeka, Tel-Aviv Academic College of Engineering

We study approximation of set-valued functions (SVFs, multifunctions) that map a compact interval $[a, b]$ into the space of compact nonempty subsets of $\mathbb{R}^d$.

Older approaches investigated almost exclusively approximation of SVFs with convex values. The standard techniques used for work with SVFs include Minkowski linear combinations and Aumann integral. These techniques are known to suffer from the phenomenon called convexification. As a result, approximation methods built on them deliver approximants whose values are convex, even if the function to be approximated did not have this property. Clearly, such methods are useless when one wants to approximate a set-valued function with general, not necessarily convex values.

A pioneering work on approximation of SVFs with general values was done by Z. Artstein who constructed piecewise linear approximants by using averages of special pairs of points that are termed in later works "metric pairs". Using the concept of metric pairs, N. Dyn, E. Farkhi and A. Mokhov developed in a series of works techniques that are appropriate for work with SVFs with general, not necessarily convex values. These techniques include metric chains, metric linear combinations, metric selections and metric integral.

In this talk we discuss metric adaptation of integral approximation operators to set-valued functions of bounded variation with compact graphs. The operators are adapted by replacing the Riemann integral by the weighted metric integral. One important example is the metric Fourier approximation that is an adaptation of partial sums of trigonometric Fourier series to SVFs. On the other hand, we consider classical positive linear operators such as the Bernstein-Durrmeyer operator and the Kantorovich operator.

We show that the pointwise limit, in the Hausdorff metric, of a sequence of such operators is $F(x)$ if x is a point of continuity of the set-valued function F. If x is a jump, we show convergence to a set $A_F(x)$ that we describe in terms of metric selections of F. Under certain assumptions on F, the limit set $A_F(x)$ equals to the average of the left and the right limits of F at x, in full agreement with the case of real-valued functions.

E–mail: elena.berdysheva@uct.ac.za.

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Orthogonal Polynomials and the associated Jacobi Operator

Christian Berg
Department of Mathematical Sciences , University of Copenhagen

To a sequence of orthonormal polynomials p_n on the real line is associated a Jacobi operator $(T, D(T))$, i.e., the operator in ℓ^2 defined as the closure of the Jacobi matrix acting on the subspace of complex sequences with only finitely many non-zero terms. It is well-known that it is symmetric with deficiency indices either $(0,0)$ or $(1,1)$. The two cases correspond to the moment problem behind being either determinate or indeterminate, i.e., there is exactly one orthogonality measure or there are several and then infinitely many orthogonality measures for p_n. In the determinate case $(T, D(T))$ is self-adjoint, but not in the indeterminate case, where it has infinitely many self-adjoint extensions. We shall focus on this case, and for a complex number z we let $\mathfrak{p}_z, \mathfrak{q}_z$ denote the sequences $(p_n(z))$ and $(q_n(z))$, where q_n denote the polynomials of the second kind. These sequence are known to be square summable.

It is known that $\mathfrak{p}_z, \mathfrak{q}_z \notin D(T)$ for all $z \in \mathbb{C}$. We determine whether linear combinations of $\mathfrak{p}_u, \mathfrak{p}_v, \mathfrak{q}_u, \mathfrak{q}_v$ for $u, v \in \mathbb{C}$ belong to $D(T)$ or to the domain of the self-adjoint extensions of T in ℓ^2. The results depend on the four Nevanlinna functions of two variables associated with the moment problem. We also show that $D(T)$ is the common range of an explicitly constructed family of bounded operators on ℓ^2.

The talk is based on recent joint work with Ryszard Szwarc, Wrocław, see arXiv:2301.00586.

E–mail: berg@math.ku.dk.

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Convergence results in Orlicz spaces for sequences of max-product Kantorovich sampling operators

Lorenzo Boccali, Danilo Costarelli, Gianluca Vinti
Department of Mathematics and Computer Science, University of Perugia

In [4], the authors investigated the approximation properties of the so-called max-product Kantorovich sampling operators K_n^χ based upon general kernels in the space of uniformly continuous and non-negative bounded functions. Furthermore, approximation results with respect to the L^p norm for the involved operators have been also established by using a direct approach. As it is well known, the max-product version of Kantorovich sampling operators represents a non-linear generalization of the linear ones [1], obtained by computing the supremum (or the maximum, in the finite case) of a set of real numbers, instead of the series (or the sum for finite terms) as in the linear case. As shown in many papers (see, e.g., [5, 6, 2]), compared with their linear counterparts, max-product operators allow to obtain sharper approximation.
In this talk, we will show a recent study on the convergence properties of K_n^χ on both bounded intervals and on the whole real axis in the general setting of Orlicz spaces L^φ, which provide a natural extension of the usual L^p-spaces, $1 \leq p < +\infty$. In particular, in [3], by applying the so-called moment-type approach in its max-product version on general kernels, we establish a modular convergence theorem in L^φ for the above sampling-type operators. The latter result makes it possible to extend by a unique general approach those proved in [4], obtaining the convergence in L^p and, in addition, in several other well-known function spaces.
Finally, the theory here developed will be applied to K_n^χ based upon some specific examples of well-known kernels, such as Fejér or B-spline-type kernel, and several others.

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E–mail: lorenzo.boccali@unipg.it.

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Modeling metastatic tumor evolution, numerical resolution and growth prediction

Iulia Martina Bulai
Department of Chemical, Physical, Mathematical and Natural Sciences, University of Sassari
Maria Carmela De Bonis, Concetta Laurita, Valeria Sagaria
Department of Mathematics, Computer Science and Economics, University of Basilicata

In this talk I will present the work developed in [2], where we have introduced a generalized metastatic tumor growth model that describes the primary tumor growth by means of an Ordinary Differential Equation (ODE) and the evolution of the metastatic density using a transport Partial Differential Equation (PDE), [4]. The numerical method is based on the resolution of a linear Volterra integral equation (VIE) of the second kind, which arises from the reformulation of the ODE-PDE model, [3]. The convergence of the method is proved and error estimates are given. The computation of the approximate solution leads to solve well conditioned linear systems. Here we focus our attention on two different case studies: lung and breast cancer. We assume five different tumor growth laws, [1], for each of them, different metastatic emission rates between primary and secondary tumors, and last that the new born metastases can be formed by clusters of several cells.

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E–mail: imbulai@uniss.it.

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An algorithm for a Constrained P-spline

Rosanna Campagna, Serena Crisci, Gerardo Toraldo
Department of Mathematics and Physics, University of Campania "L. Vanvitelli", Italy
Gabriele Santin
Fondazione Bruno Kessler, Digital Society Center, Trento, Italy
Marco Viola
School of Mathematics and Statistics, University College Dublin, Ireland

Regression splines are used mainly to estimate unknown, evenly multivariate, smooth functions and predict data behavior. The basic idea of penalizing such models is to preserve them from overfitting and boundary effects. Different penalized splines exist, balancing the regression fidelity and nice differential properties. Thanks also to the wide diffusion of applications in the field of statistics, the penalized regression splines known as P-splines [1] have gained a large success that makes them attractive for further investigations and some generalizations. Constrained P-splines defined by additional suitable penalty terms, forcing the model to be monotone or positive, according to the problem's nature or the expected values' physical meaning, are available in the literature. In this framework we formulate a constraint optimization problem that, saving the original P-spline definition and properties, controls the model shape and sign, combining a dynamical selection of additional points and monotone bounds. A theoretical result relating the boundaries and data distribution to the variational properties of the B-splines is also presented and drives the selection. We test the effectiveness of the proposed technique in the context of image completion problems [2]; by assuming that images are smooth and low-rank when viewed as matrices of pixel intensities, one can use parametrizable nonnegative functions, such as polynomials or splines [3], to convert discrete data in functional form enforcing some intrinsic features (e.g. smoothness) on the recovered data.

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E–mail: rosanna.campagna@unicampania.it.

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Representation formulae for C_0-semigroups in terms of integrated means

Francesco Altomare, Mirella Cappelletti Montano
Dipartimento di Matematica, Università degli Studi di Bari Aldo Moro

Vita Leonessa
Dipartimento di Matematica, Informatica ed Economia, Università degli Studi della Basilicata

The aim of the talk is to present certain representation formulae for C_0-semigroups acting on arbitrary Banach spaces; these representation formulae are expressed in terms of limits of integrated means with respect to some given family of probability Borel measures and other parameters (see [1]). Illustrative examples and applications are also provided.

The cases where the limits hold true pointwise or uniformly on compact subintervals are discussed separately. In fact, in order to face those problems, different methods have been required: the former case has been studied by using purely functional-analytic methods (see also [2]), the latter one by involving methods arising from Approximation Theory.

Finally, we present some estimates of the rate of convergence in terms of the rectified modulus of continuity and the second modulus of continuity.

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E–mail: mirella.cappellettimontano@uniba.it.

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Hyper-parameter tuning in kernel-based partition of unity methods

Roberto Cavoretto, Alessandra De Rossi, Sandro Lancellotti, Federico Romaniello
Department of Mathematics "Giuseppe Peano", University of Torino

Partion of Unity Methods (PUMs) are mesh-free interpolation techniques [1] that enable us to reduce cost-intensive computations, when the number of scattered data is very large. PUM interpolants are given by a sum of local radial kernel approximants and weight functions, where the entire domain is decomposed into several smaller sub-domains of variable radius. As the local kernel shape parameter and the sub-domain radius are responsible for accuracy and stability of PUMs, in order to determine optimal interpolants, we propose to exploit a statistical technique, called *Bayesian Optimization* [4]. This technique is often considered in machine learning for optimization of black-box or difficult-to-evaluate functions, and in an interpolation framework it can be used to make computation of such hyper-parameters faster than traditional methods [2,3]. Numerical experiments are carried out to support and validate this study.

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E–mail: roberto.cavoretto@unito.it.

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Some recent results about the spectrum of a dual frames multiplier

Rosario Corso
Dipartimento di Matematica e Informatica, University of Palermo

We will talk about multipliers, i.e. operators consisting of analysis, multiplication and synthesis processes, in the context of frame theory. We recall that a sequence $\varphi = \{\varphi_n\}_{n\in\mathbb{N}}$ of elements of a Hilbert space $\mathcal{H}$ is a *frame* for $\mathcal{H}$ if there exist $A, B > 0$ such that

$$A\|f\|^2 \leq \sum_{n\in\mathbb{N}} |\langle f, \varphi_n\rangle|^2 \leq B\|f\|^2, \qquad \forall f \in \mathcal{H},$$

and two frames φ, ψ are said to be *dual* if the following *reconstruction formula* holds

$$f = \sum_{n\in\mathbb{N}} \langle f, \psi_n\rangle \varphi_n, \qquad \forall f \in \mathcal{H}.$$

A dual frames multiplier is defined as

$$M_{m,\varphi,\psi} f = \sum_{n\in\mathbb{N}} m_n \langle f, \psi_n\rangle \varphi_n, \qquad f \in \mathcal{H},$$

where the analysis and the synthesis are made by two dual frames ψ and φ, respectively, and $\{m_n\}$ is a complex sequence. We will talk about some results of [1, 2] concerning the spectrum $\sigma(M_{m,\varphi,\psi})$ of a dual frames multiplier and, in particular, about the individuation of some regions of the complex plane where the $\sigma(M_{m,\varphi,\psi})$ is contained and sufficient conditions for $\sigma(M_{m,\varphi,\psi})$ to be at most countable.

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E–mail: rosario.corso02@unipa.it.

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on Approximation Theory
and Applications
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Estimates of the Approximation error for Neural Network Operators

Danilo Costarelli
Department of Mathematics and Computer Science, University of Perugia

In this talk we present quantitative estimates for the error of approximation with respect to the sup-norm, as well as, to the L^p-norm, achieved by neural network (NN) operators activated by sigmoidal functions. The above estimates have been given by means of suitable moduli of smoothness of the approximated functions. Particular emphasis will be given to the very delicate case of functions belonging to L^p-spaces, $1 \leq p < +\infty$.

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E–mail: danilo.costarelli@unipg.it.

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On the numerical solution of some elliptic PDEs with Neumann boundary conditions through multinode Shepard method

Francesco Dell'Accio, Filomena Di Tommaso[1]
Department of Mathematics and Computer Science, University of Calabria

Elisa Francomano
Department of Engineering, University of Palermo

In this talk, the multinode Shepard method [1] is proposed to solve elliptic partial differential equations with Neumann boundary conditions. The method has been opportunely handled to solve different equations with various boundary conditions dealing with scattered distribution of points [2, 3]. The particular feature of the method, based on local polynomial interpolants on opportunely choosen nearby nodes [4], is a collocation matrix which is reduced in size with many zero entrances and a small condition number. Experiments in 2d domains have been performed with Neumann boundary conditions. Comparisons with the analytic solutions and the results generated with the RBF method proposed by Kansa [5] are presented referring to different distribution of points.

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E–mail: filomena.ditommaso@unical.it.

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Optimal decomposition of the RBF interpolation matrix

Nadaniela Egidi [1], *Josephin Giacomini, Pierluigi Maponi*
Department of Mathematics, School of Science and Technology, University of Camerino

It is well known that iterative methods produce an approximate solution to a given linear system $Ax = b$ after a finite number of steps that depends on a desired precision. These methods are normally used for large systems and its computational cost depends on the ones of the matrix-vector product. So that, iterative methods are especially applied when A is large and sparse for which direct solvers are slow. On the other hand, when A is a Radial Basis Function (RBF) interpolation matrix, the coefficient matrix A is dense and so the computational cost of the matrix-vector product is fundamental for the efficiency of the used iterative method.

We show that, by approximating the RBF interpolation matrix A with a suitable decomposition [2,3], the computational cost of the matrix-vector product can be reduced from $\mathcal{O}(n^2)$ to $\mathcal{O}(n \log n)$ or to $\mathcal{O}(\alpha n)$ where α depends on the accuracy of the approximation, this cost changes as the RBF used and the decomposition vary [1].

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E–mail: nadaniela.egidi@unicam.it.

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On (β, γ)-Chebyshev functions and points

Stefano De Marchi, Giacomo Elefante, Francesco Marchetti
Department of Mathematics "Tullio Levi-Civita", University of Padova

We discuss about the recently introduced class of (β, γ)-Chebyshev functions and corresponding points. They can be seen as a family of *generalized* Chebyshev polynomials and points and, therefore, they enjoy some of the nice properties of the latters. For example, the (β, γ)-Chebyshev functions are orthogonal functions in certain subintervals of $[-1, 1]$ with respect to a weighted arc-cosine measure. Moreover, we investigate the cases where they reduce to polynomials, deriving new results concerning classical Chebyshev polynomials of first kind.

More properties related to the orthogonality of the functions are presented. In particular, we show that they satisfy a Christoffel-Darboux formula and are solution of a Sturm-Liouville problem. In addition, we show also their characterization as continued fraction and we present their generating function.

We also study the behavior of the Lebesgue constants of the polynomial interpolant at these points on varying the parameters β and γ.

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E–mail: stefano.demarchi@unipd.it, giacomo.elefante@unipd.it, francesco.marchetti@unipd.it.

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A preconditioning strategy for inverse multiquadric RBF interpolation

Nadaniela Egidi, Josephin Giacomini[1]*, Pierluigi Maponi*
Department of Mathematics, School of Science and Technology, University of Camerino

Radial basis functions (RBFs) are efficient tools for the approximation of functions [1], for scattered data interpolation problems [2,3] and for solving real-world problems involving differential equations [4,5]. Concerning interpolation problems for scattered data, their solution leads to a linear system with a dense coefficient matrix. The properties of such matrix give rise to the following main difficulties in solving the corresponding linear system: i) a high computational cost when a large number of interpolation points is considered, in fact, the matrix is dense with order equal to the number of interpolation points, ii) numerical instability, because the interpolation matrix usually has a high condition number.

We deal with the numerical instability issue and we propose a preconditioning strategy for interpolation problems with the inverse multiquadric RBF. In more detail, we propose a decomposition technique for the interpolation matrix from which a preconditioner is devised by exploiting the Sherman-Morrison-Woodbury formula [6]. This preconditioning strategy and some preliminary numerical results are shown, together with some future perspectives.

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E–mail: josephin.giacomini@unicam.it.

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Opportunities and Limitations for Deep Learning in the Sciences

Philipp Grohs
University of Vienna and Austrian Academy of Sciences

In a recent effort to push modern tools from machine learning into several areas of science and engineering, deep learning based methods have emerged as a promising alternative to classical numerical schemes for solving problems in the computational sciences – example applications include fluid dynamics, computational finance, or computational chemistry.

This talk seeks to illuminate the limitations and opportunities of this approach, both on a mathematical and an empirical level. In a first part we present computational hardness results for deep learning based algorithms and find that the computational hardness of a deep learning problem highly depends on the specific norm in which the error is measured. In a second part we present a deep learning based numerical algorithm that outperforms the previous state of the art in solving the multi electron Schrödinger equation – one of the key challenges in computational chemistry.

E–mail: philipp.grohs@univie.ac.at.

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Topological Data Analysis : extracting insights from the "shape" of data

K. Hess Bellwald
Laboratory for topology and neuroscience, EPFL Lousanne (CH)

The guiding philosophy of topological data analysis (TDA) is that the "shape" of a point cloud of data should reveal meaningful structure in the data. I will briefly sketch the theoretical foundations of TDA, describe some of its most frequently used and powerful methods, and conclude with examples of significant applications in cancer biology, neuroscience, and material science.

https://www.epfl.ch/labs/hessbellwald-lab/hessbellwald/

E–mail: kathryn.hess@epfl.ch.

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Birkhoff–Kellogg type result in cones with applications

Gennaro Infante
Dipartimento di Matematica e Informatica, Università della Calabria, 87036 Arcavacata di Rende, Cosenza, Italy

We present some classical and modern results of Birkhoff–Kellogg type and their application to the solvability of parameter-dependent problems in differential equations.

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E–mail: gennaro.infante@unical.it.

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On the solvability of some boundary integral equations of the first kind and applications

Alberto Cialdea, , Vita Leonessa, Angelica Malaspina
Department of Mathematics, Computer Science and Economics, University of Basilicata, Italy

The aim of this talk is to focus on a method for solving integral equations of the first kind that naturally arise in the problem of representability of the solutions to some boundary value elliptic differential problems in terms of simple or double layer potentials. The integral representations obtained are simpler than the ones obtained by the direct methods, especially for higher order equations. This could lead also to advantages in numerical applications (see, e.g. [5]).

Coming back to the method used, it was introduced by Cialdea in [1] and subsequently applied to other differential problems (see, e.g. [2,3,4]). Its simplest application case is the following one.

Consider the Dirichlet problem for the Laplace equation

$$\begin{cases} \Delta u = 0 & \text{in } \Omega\,, \\ u = g & \text{on } \Sigma = \partial\Omega\,, \end{cases}$$

$\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ being a bounded simply connected domain and $g \in W^{1,p}(\Sigma)$ $(1 < p < +\infty)$, and suppose that we are interested to represent its solution in the form of a simple layer potential with density $\varphi \in L^p(\Sigma)$.

By imposing the boundary condition, we get the integral equation of the first kind on Σ

$$\int_\Sigma \varphi(y) s(x,y)\, d\sigma_y = g(x)\,, \quad x \in \Sigma, \tag{1}$$

where $s(x,y)$ is the fundamental solution of the Laplace equation. The method proposed consists in taking the differential of both sides in (1) and studying the related singular integral equation

$$S\varphi(x) := \int_\Sigma \varphi(y) d_x[s(x,y)]\, d\sigma_y = dg(x)\,, \quad x \in \Sigma \tag{2}$$

in which the unknown is a scalar function while the data is a differential form of degree 1. Thanks to the fact that the operator S can be reduced on the left, it can be showed that equation (2) is equivalent to a Fredholm one which is always solvable.

We shall see that such method can be extended to different boundary value problems associated with partial differential equations and systems both in simply and multiple connected domains.

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E–mail: vita.leonessa@unibas.it.

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Using annihilation operators to combine several linear subdivision schemes into a single non-linear one

Rosa Donat, Sergio López-Ureña[1]
Department of Mathematics, University of Valencia

The reproduction of trigonometric functions is a valuable attribute for subdivision schemes in fields such as CAGD. The reproduction of exponential polynomials, a more general space, has been thoroughly investigated [1, 2] and can be accomplished using linear non-stationary subdivision schemes.

A unique four-point linear and non-stationary subdivision scheme exists that can reproduce

$$\mathcal{F}(\gamma) := \operatorname{span}\{1, \exp(i\gamma t), \exp(-i\gamma t)\} = \operatorname{span}\{1, \cos(\gamma t), \sin(\gamma t)\}, \qquad i = \sqrt{-1},$$

for any given $\gamma \in (-\pi, \pi) \cup i\mathbb{R}$, $\gamma \neq 0$. However, to define this subdivision scheme, the value of γ must be known. In this talk, we present a method for defining a four-point non-linear and stationary subdivision scheme that can reproduce $\mathcal{F}(\gamma)$ without depending on the value of γ. The key idea here is to use annihilation operators [1, 3]. As a result, this non-linear subdivision scheme can reproduce $\mathcal{F}(\gamma)$ for all $\gamma \in (-\pi, \pi) \cup i\mathbb{R}$.

In addition, we proved [4] that this scheme is convergent and maintains monotonicity. In monotone areas, it has a fourth-order approximation, is stable and has $\mathcal{C}^1$ regularity. This technique can be extended to the bivariate case as well [5].

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E–mail: sergio.lopez-urena@uv.es.

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Mapped variably scaled kernels and applications

Francesco Marchetti, Stefano De Marchi
Department of Mathematics, University of Padova

Emma Perracchione
Department of Mathematics, Polytechnic of Torino

Anna Volpara, Anna Maria Massone, Michele Piana
Department of Mathematics, University of Genova

Variably scaled kernels [1] and mapped bases constructed via the so-called fake nodes approach [2] are two different strategies to provide adaptive bases for function interpolation. In this talk, we present what we call mapped variably scaled kernels, which take advantage of both strategies. We present some theoretical analysis and then we show their efficacy via numerical experiments and in concrete applications.

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E–mail: francesco.marchetti@unipd.it.

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A Galerkin-type method for Fredholm integral equations over equispaced nodes

Domenico Mezzanotte, Donatella Occorsio
Department of Mathematics, Computer Science and Economics, University of Basilicata
Francesco Dell'Accio, Federico Nudo
Department of Mathematics and Computer Science, University of Calabria

In this talk, we present a Galerkin-type method based on equally spaced points of $[-1,1]$, for Fredholm integral equations (FIEs) of the second kind

$$f(y) - \mu \int_{-1}^{1} f(x)k(x,y)w(x)dx = g(y), \quad y \in [-1,1],$$

where f is the unknown function, k is a bivariate function defined in the unit square $[-1,1]^2$ that can also contain some peculiar drawbacks, g is the right-hand side term, $\mu \in \mathbb{R}$, and $w(x) = v^{\alpha,\beta}(x) := (1-x)^\alpha(1+x)^\beta$, $\alpha, \beta > -1$ is a Jacobi weight.

The use of equidistant points is crucial in many engineering and mathematical physics problems which are modeled by integral equations, and when k and g are available only in a discrete set of equispaced nodes as results of experiments on the field. In all these cases, the classical methods based on piecewise polynomial approximation offer a lower degree of approximation, while the efficient procedures based on the zeros of orthogonal polynomials cannot be used.

Here, we present a Galerkin method based on quadrature formulae (see, e.g. [1,2]) obtained by means of the constrained mock-Chebyshev least squares operator $\hat{P}_{r,n}$. This operator was recently introduced in order to defeat the Runge phenomenon that occurs when using polynomial interpolation on large sets of equally spaced points [3].

Stability and convergence of the method are proved in suitable spaces and some numerical tests are shown to confirm the theoretical estimates.

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E–mail: domenico.mezzanotte@unibas.it.

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General methods for enriching the simplicial linear finite elements

Francesco Dell'Accio, Filomena Di Tommaso, Federico Nudo [1]
Department of , University of Mathematics and Computer Science, University of Calabria

Allal Guessab, Federico Nudo
Laboratoire de Mathématiques Appliquées de Pau (LMAP) - UMR 5142, Université de Pau et des Pays de l'Adour

Low-order elements are widely used and preferred for finite element analysis, specifically the three-node triangular and four-node tetrahedral elements, both based on linear polynomials in barycentric coordinates. They are known, however, to under-perform when nearly incompressible materials are involved. The problem may be circumvented by the use of higher degree polynomial elements, but their application become both more complex and computationally expensive. For this reason, non-polynomial enriched finite element methods have been proposed for solving engineering problems. In line with previous researches [1,2], the main contribution of this work is to present a general strategy for enriching the standard simplicial finite element by non-polynomial functions. A key role is played by a characterization result, given in terms of the non-vanishing of a certain determinant, which provides necessary and sufficient conditions, on the enrichment functions and functionals, that guarantee the existence of families of such enriched elements. We show that the enriched basis functions admit a closed form representation in terms of enrichment functions and functionals. Finally, we provide concrete examples of admissible enrichment functions and perform some numerical tests.

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E–mail: federico.nudo@unical.it.

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On the regularization properties of Durrmeyer-sampling type operators in L^p-spaces

Danilo Costarelli[a], *Michele Piconi*[a,b], *Gianluca Vinti*[a]
[a] Department of Mathematics and Computer Science, University of Perugia
[b] Department of Mathematics and Computer Science "Ulisse Dini", University of Florence

In [4], we started to study the approximation properties of Durrmeyer-sampling type operators [6, 2] in the general setting of Orlicz spaces, a natural extension of Lebesgue spaces, with particular attention to the problem of convergence.
As it is well-known, Durrmeyer operators $D_w^{\varphi,\psi}$ are based upon two kernels φ and ψ and originate from a sharp modification of Bernstein polynomials, aimed at extending the celebrated Weierstrass approximation theorem to more general functional spaces, so that to include also not necessarily continuous functions. This is especially useful in the signal and image processing, where most of the real-world signals may present jump discontinuities (such as digital images). Moreover, as shown in [4], the above operators extend other well-known families of operators, such as generalized [3] and Kantorovich-sampling type operators [1].
In this talk, I will show a recent study on the regularization properties of $D_w^{\varphi,\psi}$ in L^p-spaces $1 \leq p \leq +\infty$, carried out by using a distributional approach. In particular, we would like to highlight how the regularization process performed by the operators on the function is strongly influenced by the regularity of the discrete kernel φ. In [5], we investigate the classical case when φ is a continuous function, the more general case when it belongs to Sobolev spaces, as well as the remarkable case when it is bandlimited, i.e., belonging to a Bernstein class. In the latter case, we also establish a closed form for the distributional Fourier transform of the above operators applied to bandlimited functions.
Finally, the main results presented herein will be also discussed via specific instances of bandlimited kernels, such as Fejér or Bochner-Riesz kernel.

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E–mail: michele.piconi@unifi.it.

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Fixed point index theory for compositions of usc multivalued maps

Jorge Rodríguez-López [1]
CITMAga, University of Santiago de Compostela

The fixed point index in cones and the fixed point theorems derived from it (in particular, compression-expansion ones) are between the main tools in the study of the existence, localization and multiplicity of solutions for large classes of boundary value problems. Here, we present a new fixed point index theory for the so-called decomposable maps (that is, compositions of two upper semicontinuous multivalued maps). Of course, it allows to extend to this class of maps a large number of fixed point theorems which are well-known in the context of compact operators.

As an application of the theory, we prove new existence results for ϕ-Laplacian problems with discontinuous nonlinearities and nonlinear functional boundary conditions.

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E–mail: jorgerodriguez.lopez@usc.es.

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On the construction of compactly supported fundamental functions for interpolation via polynomial blends

Carolina Vittoria Beccari, Giulio Casciola, Lucia Romani
Department of Mathematics, University of Bologna

In this talk we discuss a strategy that exploits polynomial blends to construct compactly supported fundamental functions for local interpolation.
In the univariate case, we specialize the results introduced in [1] by assuming that the blending functions used to combine suitably constructed polynomial interpolants are always piecewise-polynomial functions with compact support and, in particular, cubic B-splines. The resulting fundamental functions for interpolation will be always globally C^2 at least, but, in some circumstances, they could also reach C^3 smoothness everywhere.
In the bivariate case, the fundamental functions for local interpolation are assumed to be defined over quadrangulations with isolated extraordinary vertices. Their construction consists in considering a combination of bivariate polynomial interpolants with blending functions that are either tensor-products of cubic B-splines or basic limit functions of the Catmull-Clark subdivision scheme [2]. Differently from the univariate case, the blending functions used in the bivariate construction are always globally C^1 since, although they are usually smoother than C^1 in the regular regions, they are C^1 at most at extraordinary vertices. However, thanks to our construction, the resulting fundamental functions for interpolation will be always globally C^2.

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E–mail: lucia.romani@unibo.it.

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A Moment for Multivariate Continued Fractions

Tomas Sauer
Chair of Mathematical Image Processing & FORWISS , University of Passau, Fraunhofer IIS, Department EZRT

Yuan Xu
Department of Mathematics, University of Oregon

In a single variable, moment sequences, Gauss quadrature and continued fractions are closely related; in fact, many of the concepts turn out to be equivalent. In several variables, the situation is significantly different: there is no concept of of continued fractions, orthogonal polynomials exist, but are more intricate, thre-term recurrences have a complex structure and the existence of Gaussian cubature is a rarity.

By relating all these concepts and intertwining them with Pronys problem, we come up with a concept of multivariate continued fractions as rational approximations of formal Laurent series that relate Gaussian quadrature to the flat extension of moment sequences.

E–mail: tomas.sauer@uni-passau.de.

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Generalizing Floater - Hormann rational interpolation

Woula Themistoclakis [1]
CNR - IAC Institute for Applied Computing "Mauro Picone"
Marc Van Barel
Department of Computer Science, KU Leuven

As known, polynomial interpolation is not advisable in the case of equidistant nodes, given the exponential growth of the Lebesgue constants and the consequent stability problems.

In [1] Floater and Hormann introduce a family of rational interpolants (briefly FH interpolants) depending on a fixed integer parameter $d \geq 1$. They are based on any configuration of the nodes in $[a, b]$, have no real poles and approximation order $O(h^{d+1})$ for functions in $C^{d+2}[a, b]$, where h denotes the maximum distance between two consecutive nodes.

FH interpolants turn out to be very useful for equidistant or quasi-equidistant configurations of nodes when the Lebesgue constants present only a logarithmic growth as the number of nodes increases [2, 3].

In this talk, we introduce a generalization of FH interpolants depending on an additional parameter $\gamma \in \mathbb{N}$. If $\gamma = 1$ we get the classical FH interpolants, but taking $\gamma > 1$ we succeed in getting uniformly bounded Lebesgue constants for quasi-equidistant configurations of nodes. Moreover, in comparison with the original FH interpolants, we show that the new interpolants present a much better error profile when the function is less smooth.

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[1] Aknowledgements...
E–mail: woula.themistoclakis@cnr.it.

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A Mathematical Model for the Study of Vascular Pathologies

Arianna Travaglini[1,2], *Gianluca Vinti*[1]

[1]Department of Mathematics and Computer Science, University of Perugia, 1, Via Vanvitelli, 06123 Perugia, Italy

[2] Department of Mathematics and Computer Science "U.Dini"-DIMAI, University of Florence, 67/a, Viale Giovanni Battista Morgagni, 50134 Firenze, Italy

Among sampling-type operators, the sampling Kantorovich operator represents a useful tool for dealing with not necessarily continuous functions ([1]). Its muldimensional version ([5]) has been implemented and allows not only to reconstruct, but also to increase the information content of images, as it acts both as a low-pass filter and as a rescaling algorithm ([2]). Indeed, the sampling Kantorovich algorithm has been used, with satisfactory results, in e.g., [3,4,6].
The talk is focused on some recent results ([6]) which constist in a study developed on 13677 images from 15 patients affected by moderate/severe atheromatous disease of the abdominal aortic tract. A procedure to extract the pervious lumen of the aorta artery from basal CT images is exploited ([3]), implemented in a portal, namely ImageLab, and tested on a large scale. Taking benefits from its reconstruction and enhancing properties, the sampling Kantorovich algorithm is used to process the considered images, which are compared, slice by slice, by registration, with the corresponding contrast medium reference image.
Numerical indices of errors were computed and analyzed, together with a clinical evaluation, in order to test the validity of the proposed method. In order to make this study useful for an assisted diagnosis of the pathologies examined, the so called "Care portal", which has been projected and implemented to allow the interaction between doctors and mathematicians, is also described.

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E–mail: arianna.travaglini@unifi.it.

International Conference
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Cetraro, 18–22 June 2023

Analytic Continuation, Rational Approximation, and Laplace Problems

L. N. Trefethen
Oxford University Mathematical Institute

In many applications the best known method for numerical analytic continuation is AAA rational approximation. We show numerical experiments, including the "one-wavelength principle", and review relevant theory including the notion of the Schwarz function. Analytic continuation leads to estimates of poles and branch points near boundaries, and from here, following the AAA-Least Squares idea first proposed by Stefano Costa, one derives a remarkably fast and accurate method for solving planar Laplace problems.

E–mail: trefethen@maths.ox.ac.uk.

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Sigmoidal functions and multiscale resolution of singularities

L. N. Trefethen
Oxford University Mathematical Institute

D. Huybrechs
Department of Computer Science of KU Leuven

We show that essentially the same mathematics applies in three contexts with disparate literatures:

(1) sigmoidal and RBF approximation of smooth functions,
(2) rational approximation of analytic functions near singularities,
(3) hp mesh refinement for solution of PDEs.

The relationship of (1) and (2) is as simple as the change of variables s = log(x), and our informal mnemonic for this relationship is "sigmoid = log(ratapprox)".

E–mail: trefethen@maths.ox.ac.uk.

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Univariate Dual Interpolating Subdivision: Characterization, Construction and Implementation

Alberto Viscardi
Department of Mathematics "Giuseppe Peano", University of Turin

In the classical univariate functional setting, a subdivision scheme is a linear iterative process that, given an initial set of values over $\mathbb{Z}$, converges to a continuous limit which is a linear combination of the shifts of a unique function φ. This particular function, that characterizes a scheme, is continuous, compactly supported and it satisfies a refinement equation involving a *dilation factor* $m \in \mathbb{N} \setminus \{1\}$, a sequence $\mathbf{a} = [a_k \in \mathbb{R}]_{k\in\mathbb{Z}}$ and a *shift parameter* $\tau \in \{0, 1/2\}$. A scheme with $\tau = 0$ is usually called *primal*, while $\tau = 1/2$ defines the so called *dual* schemes.

It is well known that a sufficient condition for a scheme to be interpolating is the *step-wise interpolation property*, which forces the scheme to be primal ($\tau = 0$) and to have the Kronecker delta $\boldsymbol{\delta} = [\delta_{0,k}]_{k\in\mathbb{Z}}$ as a specific sub-sequence of $\mathbf{a}$. This condition is also known to be necessary when $m = 2$. What happens then for $m > 2$ and $\tau = 1/2$? The aim of this talk is to answer this question systematically.

First, the existence of a simple dual interpolating scheme will be shown. Then an algebraic characterization of dual interpolating schemes will be provided in terms of the *symbols* of a scheme (special Laurent polynomials whose coefficients depend on $\mathbf{a}$ and on certain samples of φ). This characterization divides dual interpolating schemes into two sub-classes, based on m being odd or even, and it can be used to construct subdivision schemes of this kind having prescribed properties, e.g., a certain degree of polynomial reproduction. Lastly, a further investigation shows that dual interpolating schemes with an odd m have a 2-*step interpolation property*, while an even m corresponds to a complete *non-step-wise interpolation*. These facts can be exploited to optimize the numerical implementation of such schemes.

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E–mail: alberto.viscardi@unito.it.

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Some Results on approximation with nonlinear operators in Orlicz spaces

Gianluca Vinti , Luca Zampogni
Department of Mathematics and Computer Sciences, University of Perugia, Italy

We discuss certain properties of the approximation with nonlinear operators in Orlicz spaces $L^{\varphi}(G)$ with respect to modular convergence, for functions defined in a locally compact topological group G. We will focus on some fundamental assumptions which are needed to guarantee the convergence, and discuss the main result of modular convergence in a subset $\mathcal{Y} \subset L^{\varphi}(G)$.

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E–mail: luca.zampogni@unipg.it.

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Umbral quadrature formulas (Poster)

Francesco Aldo Costabile, Maria Italia Gualtieri, Anna Napoli
Department of Mathematics and Computer Science, University of Calabria

We present a new quadrature procedure based on umbral interpolation. The quadrature formulas are obtained by replacing the integrand function with an interpolating umbral polynomial.

Let $P_n \subset X,\ n \in \mathbb{N}$, be the space of polynomials of degree $\leq n$, $X = C^k[a,b]$. Let L be a linear functional on X with $L(1) \neq 0$ and Q a δ-operator on P_n, that is, $Q[y] = \sum_{k=1}^{\infty} c_k y^{(k)}, c_1 \neq 0$, $\forall y \in X$. We define $X^{Q,n} = \left\{ f \in X \,:\, Q^i[f] \in X,\ i = 0, \ldots, n \right\},\ \forall n \in \mathbb{N}$.

For every $f \in X^{Q,n}$, the unique polynomial $p_n[f]$ of degree $\leq n$ such that

$$L(Q^i[f]) = L(Q^i[p_n[f]]), \quad i = 0, 1, ..., n.$$

is the *umbral interpolant* for the function f related to the functional L and the δ-operator Q.

It can be written as

$$p_n[f](x) = \sum_{i=0}^{n} \frac{L(Q^i[f])}{i!} s_i(x),$$

where $\left\{ s_k \right\}_{k \in \mathbb{N}}$ is the Sheffer sequence related to (L, Q), called umbral basis.

If Q is the δ-operator of differentiation $Q = \frac{d}{dx} = D$, we obtain the Appell family of umbral interpolant and hence the *Appell quadrature formula*

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n} \frac{L\left(f^{(i)}\right)}{(i+1)!} \left[a_{i+1}^L(b) - a_{i+1}^L(a) \right], \qquad \forall n \in \mathbb{N},$$

where $\left\{ a_{L,i} \right\}_{i \in \mathbb{N}}$ is the Appell p.s. defined by the functional L.

For suitable choices of the functional L we get classic quadrature formulas.

Related to the interpolant polynomials there are the so-called *complementary* interpolant polynomials, from which other quadrature formulas are derived.

Better approximations are achieved by using composite formulas. New and known composite quadrature formulas are given. For example, the classic Euler-McLaurin formula is obtained from the general theory.

A numerical estimation of the remainder are given. Some numerical examples are provided.

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E–mail: anna.napoli@unical.it.